

EXPERIMENTAL STUDY OF TURBULENT FLOWS OF WEAK POLYMER SOLUTIONS IN A PIPE

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Experimental data are presented on the frictional drag of weak solutions of guaiacum, polyox, and polyacrylamide flowing in pipes, as well as on the measured velocity profiles in guaiacum solutions. The mixing lengths in the pipe cross section are calculated.

Numerous experimental investigations have shown that when small amounts (hundredths and thousandths of a percent) of certain high-molecular substances are dissolved in water the coefficient of friction in turbulent flow is appreciably reduced. The mechanism of this effect is still not fully clear. An attempt can be made, however, to predict the magnitude of the observed effect as a function of the type of polymer and its concentration in solution. Within this framework, papers by Meyer [1] and Elata, et al. [2] deserve attention. These authors proposed a semiempirical relation between the drag and the Reynolds number of a turbulent flow of polymer solutions in a pipe; their relation is analogous to the corresponding relation for water, except for the introduction of two additional parameters. One parameter characterizes the time of inception of the drag-reduction effect, and the second characterizes the magnitude of that effect.

The present study was undertaken in order to test the relations proposed in [1, 2]. We performed three series of experiments with aqueous solutions of guaiacum resin, polyethylene oxide (polyox), and polyacrylamide having a mol. wt. $M = 3.9 \cdot 10^6$, to investigate the average characteristics of their turbulent flow in a pipe.

Description of the Experimental Setup

We investigated the flow of weak polymer solutions in a closed hydraulic system driven by pumps having a maximum volumetric rate of 15 liters/sec. The hydraulic system, with a 450 liter capacity, consisted of two tanks, one for pressurization and one for discharge, which were interconnected by a working duct 35.5 mm in diameter and 4.5 m in length.

The volumetric flow rate was measured according to the static pressure drop between the pressure tank and the initial section of the working duct, which was calibrated with a magnetic-induction flowmeter.

The velocity profile over the tube cross section and the frictional drag of the fluid flow in the tube were measured in cross sections situated more than 100 diameters from the exit site of the tube from the tank, where the flow was assumed to be fully developed. The velocity profile was measured with a total-head microtube with intake orifice dimensions of 0.5×2 mm. A coordinate positioning device was used to move the microtube about in the tube cross section, with a position error of ± 0.05 mm. The position of the intake orifice relative to the wall was also determined with an error of ± 0.05 mm.

All the pressure-drop measurements were carried out with mercury-water and air-water manometers, which yielded errors of 1 or 2% in measurements of the drag, flow rate, and velocity profile in the tube cross section.

The Reynolds numbers in the experiments ranged from $2 \cdot 10^4$ to $2 \cdot 10^5$.

Preparation of the Solutions and Experimental Procedure

We carried out three series of tests to study the turbulent flow of aqueous solutions of the following polymers in tubes: guaiacum resin in concentrations by weight of $7.5 \cdot 10^{-5}$, $1.8 \cdot 10^{-4}$, $3.6 \cdot 10^{-4}$, and

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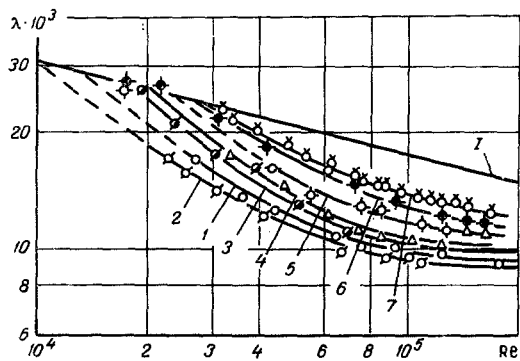


Fig. 1

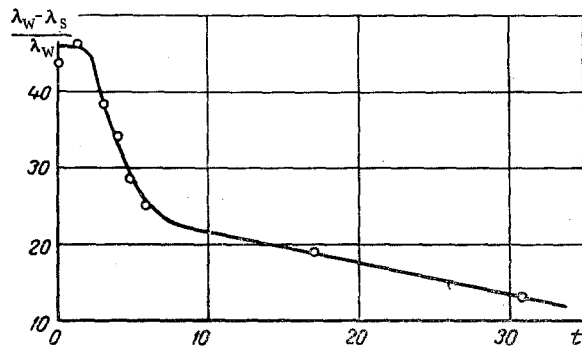


Fig. 2

Fig. 1. Drag λ versus Reynolds number Re for pipe flows of water (I) and a polyox solution of concentration $c = 10^{-6}$ g/cm³ at the following solution preparation times: 1) 0.1; 2) 1.5; 3) 3; 4) 4; 5) 6; 6) 17; 7) 31 h.

Fig. 2. Relative drag reduction $(\lambda_W - \lambda_S) / \lambda_W$, %, in the flow of a polyox solution of concentration $c = 10^{-6}$ g/cm³ at $Re = 5 \cdot 10^4$ versus solution preparation time, t , h.

6.5 · 10⁻⁴ g/cm³; polyox in concentrations of 10⁻⁶, 5 · 10⁻⁶, 10⁻⁵, 1.4 · 10⁻⁵, 2.8 · 10⁻⁵, and 5 · 10⁻⁵ g/cm³; polyacrylamide in concentrations of 1.5 · 10⁻⁴ and 3 · 10⁻⁴ g/cm³.

Due to the impossibility of preparing low-concentration solutions within the volume of our apparatus (450 liters), we prepared the solution at five to 10 times the desired concentration in a smaller volume. This concentrated solution was poured into the apparatus, which had been previously filled with pure water, and was mixed at a very slow speed for 5 min (the time required for double traversal of the liquid from the discharge tank into the pressure tank), whereupon it was considered to be ready for testing.

The concentrated polyacrylamide solutions were prepared from pellets, which were completely dissolved after only a few days.

The polyox and guaiacum solutions were prepared from powders. The following methodological procedure was followed with the polyox solutions in order to decide the proper choice of solution preparation time.

Several batches of the polyox solutions were prepared in a 10 liter volume at a concentration of 5 · 10⁻⁵ g/cm³. The time elapsed from the preparation of the solution until testing (keeping time) was varied from zero (i.e., the ready solution was tested immediately) to 31 h. Then this solution was diluted to a concentration of 10⁻⁶ g/cm³ in the apparatus, whereupon the drag was determined as a function of the pipe-flow Reynolds number for this solution. The drag data are given in Fig. 1. It is evident from the graph that as the keeping time is increased the solution "ages," i.e., its effectiveness diminishes. The quantitative assessment of this effect is illustrated by the graph of Fig. 2, in which the values of the drag for $Re = 10^5$ are plotted as a function of the keeping time for the tested solution. It is seen from the graph that the polyox solution of concentration 10⁻⁶ g/cm³ is the most effective in the first 2 h, after which its effectiveness falls off drastically. In this particular investigation we noted a 3.5-fold reduction in effectiveness after 31 h.

On the basis of these data all the subsequent investigations of polyox solutions were carried out no later than 2 h after their preparation.

The maximum keeping time for the concentrated guaiacum solutions was also limited to 2 h.

The viscosity of the solution was determined for each test by means of an Ubbelohde-type capillary viscosimeter with a capillary diameter of 0.54 mm. The polyox solutions having the investigated range of concentrations exhibited a maximum discrepancy of 3% in the viscosity, hence the viscosity of these solutions was assumed to be equal to the viscosity of water at the given temperature.

The curves presented throughout the article for flows of polyacrylamide and guaiacum solutions were obtained in experiments using a single batch of the solution, whereas the curves for the polyox solutions were obtained in experiments using 10 to 15 batches, and a total of five or six points was recorded for

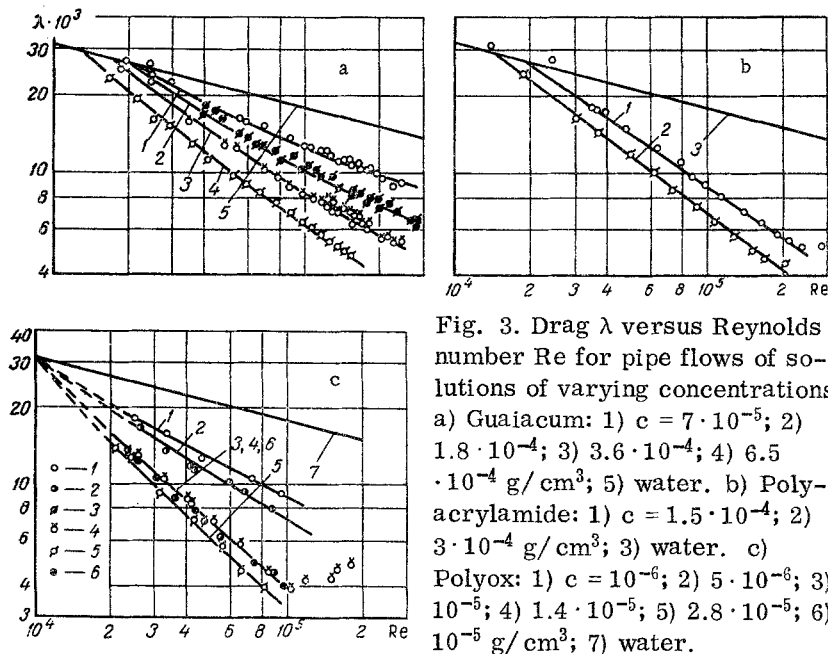


Fig. 3. Drag λ versus Reynolds number Re for pipe flows of solutions of varying concentrations. a) Guaiacum: 1) $c = 7 \cdot 10^{-5}$; 2) $1.8 \cdot 10^{-4}$; 3) $3.6 \cdot 10^{-4}$; 4) $6.5 \cdot 10^{-4}$ g/cm³; 5) water. b) Polyacrylamide: 1) $c = 1.5 \cdot 10^{-4}$; 2) $3 \cdot 10^{-4}$ g/cm³; 3) water. c) Polyox: 1) $c = 10^{-6}$; 2) $5 \cdot 10^{-6}$; 3) 10^{-5} ; 4) $1.4 \cdot 10^{-5}$; 5) $2.8 \cdot 10^{-5}$; 6) 10^{-5} g/cm³; 7) water.

a single batch of the solution. In view of the good reproducibility of the tests, only the data for one batch are represented in the graphs.

After each investigation of the flow of a solution the pipe was washed twice and the purity of its surface tested according to the agreement of the experimental data with the known relations for the frictional drag of water.

Pipe-Flow Resistance (Drag) of the Tested Solutions

The experimental data on the measured drag of turbulent pipe flows of the investigated polymer solutions were represented in the form of the drag λ as a function of the Reynolds number Re .

The data for the guaiacum and polyacrylamide solutions are shown in the graphs of Fig. 3a and 3b, respectively. The behavior of the dependences is exactly identical in each case. When a certain value of Re is attained, corresponding to a particular stress threshold, the drag of the solution flow is seen to diminish relative to water as the Reynolds number is increased, this diminution becoming more marked as the Re value and solution concentration are increased. A stratification of the threshold Reynolds numbers as a function of the solution concentration is observed; the threshold value of Re is smaller, the higher the concentration. It was noted during the experiments that there were no stable readings of the drag at the maximum attainable volumetric flow rates in the experiment; it was observed to increase slowly but steadily. Upon reversal, the resulting dependence of the drag on the Reynolds number was not repeated.

The dependence of the drag on the Reynolds number for flows of polyox solutions is shown in Fig. 3c for the entire investigated range of concentration measurements. The presence of a critical frictional stress ($\tau = 150$ to 200 dyn/cm²) is clearly pronounced on this graph, corresponding to a Reynolds number $Re = 10^5$ to $1.2 \cdot 10^5$; above this critical value a growth of the drag is observed.

The flows of polyox solutions exhibit certain peculiarities by comparison with the flows of guaiacum and polyacrylamide solutions, namely a constant threshold stress at all concentrations ($\tau_* = 5$ dyn/cm²) and the emergence of an optimum concentration ($c_{opt} = 2.8 \cdot 10^{-5}$ g/cm³), at which the effect becomes saturated, any further increase in the concentration failing to bring about an increase in the net effect as long as all other conditions are held equal.

Flow-Velocity Profiles for Polymer Solutions in a Tube

The velocity profiles measured in guaiacum solutions of various concentrations at equal volumetric flow rates $q = 6.0$ liters/sec are shown in Fig. 4 in semilog coordinates. It was impossible to measure the velocity profile in solutions having concentration $c = 6.5 \cdot 10^{-4}$ g/cm³, due to a "defect" of the pipe, namely the fact that the microtube registered a total pressure from 10 to 20% lower than the true value, an

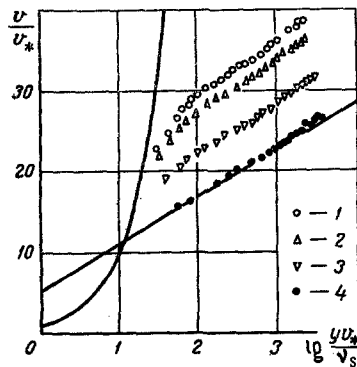


Fig. 4

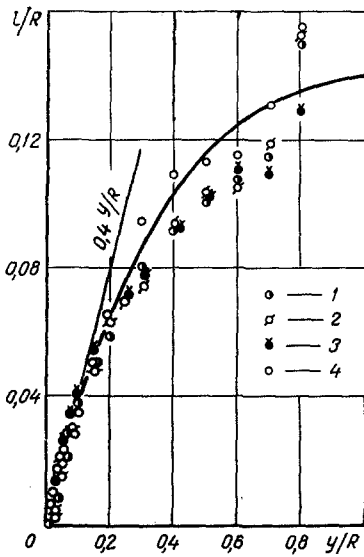


Fig. 5

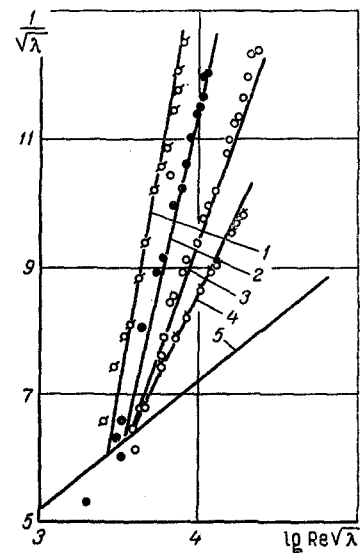


Fig. 6

Fig. 4. Velocity distribution v/v_* in the pipe cross section for flows of guaiacum solutions of various concentrations. 1) $c = 3.6 \cdot 10^{-4}$; 2) $1.8 \cdot 10^{-4}$; 3) $7.4 \cdot 10^{-5}$ g/cm³; 4) pure water.

Fig. 5. Mixing length l/R over the pipe cross section y/R for flows of guaiacum solutions of various concentrations. 1) $c = 3.6 \cdot 10^{-4}$; 2) $1.8 \cdot 10^{-4}$; 3) $7.4 \cdot 10^{-5}$ g/cm³; 4) pure water.

Fig. 6. Graph of $l/\sqrt{\lambda}$ vs $\log(\text{Re}\sqrt{\lambda})$ for guaiacum solutions of various concentrations flowing in a pipe. 1) $c = 6.5 \cdot 10^{-4}$; 2) $3.6 \cdot 10^{-4}$; 3) $1.8 \cdot 10^{-4}$; 4) $7.4 \cdot 10^{-4}$ g/cm³; 5) pure water.

effect that was reflected in the disparity of the flow rates determined with the flowmeter and calculated on the basis of the measured profile. This effect did not appear for smaller values of the concentration, and the flow rate of the liquid in the pipe cross section, as calculated from the measured profiles, agreed with the flowmeter readings correct to 0.5 to 1%. It is evident from the graph that the measured velocity profiles for the guaiacum solutions have the same slopes as the profiles for water in coordinates of v/v_* vs $\log(yv_*/\nu_s)$.

The measured velocity profiles and corresponding drag coefficients at the wall were used to calculate the mixing lengths (Fig. 5). Due to the considerable error in the determination of the derivative of the average velocity profile in the center of the pipe a certain scatter is observed in the mixing length in this region. It is noted, however, that the mixing length in the solutions scarcely differs from the value for water within the error limits of the calculations.

Analytical Description of the Experimental Results

We seek to describe the experimental data from our measurements of the average turbulent-flow characteristics of the solutions in the region where the drag-reduction effect is observed to increase with the velocity.

We have borrowed the equations of [1, 2] in order to determine the required dependence.

The relation given in [1] between the drag coefficient and the Reynolds number for the flow of solutions in pipes represents a straight line in the coordinates $1/\sqrt{\lambda}$ vs $\log(\text{Re}\sqrt{\lambda})$:

$$\frac{1}{\sqrt{\lambda}} = \left(2 + \frac{\alpha_M}{\sqrt{8}}\right) \lg \text{Re}\sqrt{\lambda} - 0.8 - \frac{\alpha_M}{\sqrt{8}} \lg \frac{\sqrt{8} dv_{*t}}{\nu_s}, \quad (1)$$

where v_{*t} and α_M are parameters.

Meyer [1] postulated that the parameter v_{*t} characterizing the threshold stress is universal, assuming it to be equal to 0.07 m/sec on the basis of experiments with carboxymethylcellulose (CMC) solutions flowing in a tube. The coefficient α_M , which characterizes the magnitude of the drag-reduction effect, depends on the solution concentration and the type of polymer.

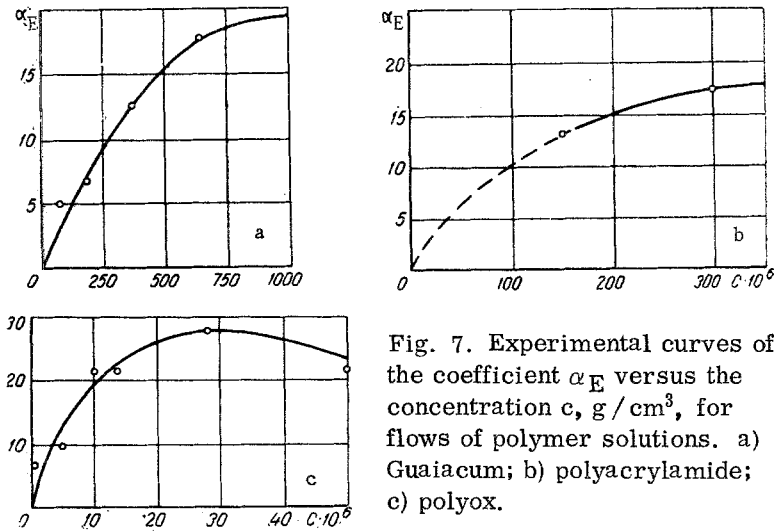


Fig. 7. Experimental curves of the coefficient α_E versus the concentration c , g/cm^3 , for flows of polymer solutions. a) Guaiacum; b) polyacrylamide; c) polyox.

It must be pointed out, however, that the universality of the parameter v_{*t} is somewhat questionable, because the experimental results described in the present article show that this quantity has different values, depending on the type of polymer and, sometimes, on the concentration of the solution. The following relation, analogous to (1), is proposed in [2]:

$$\frac{1}{\sqrt{\lambda}} = \left(2 + \frac{2\alpha_E}{\sqrt{8}} \right) \lg \operatorname{Re} \sqrt{\lambda} - 0.8 + \frac{\alpha_E}{\sqrt{8}} \lg \frac{\tau_1 v_s}{8d^2}. \quad (2)$$

In the coordinates $1/\sqrt{\lambda}$ vs $\lg(\operatorname{Re}\sqrt{\lambda})$ this characteristic describes a straight line whose slope relative to the axes of the plot differs from its analog for water. The parameters τ_1 and α_E involved in this equation have the same meaning as v_{*t} and α_M .

The parameter τ_1 (relaxation time) is not determined experimentally, but is found on the basis of the physicochemical properties of the polymer (molecular weight, solution viscosity, concentration, and temperature), i.e., it is to be interpreted as the relaxation time for the polymer molecule in weak solution:

$$\tau_1 = \frac{6\rho v}{\pi^2 kTN} \frac{v_s - v}{vc} M. \quad (3)$$

As shown in [2], the relaxation time can be expressed in terms of the threshold values of the Reynolds number and the drag coefficient:

$$\operatorname{Re}_t \sqrt{\lambda}_t = d \sqrt{\frac{8}{v_s \tau_1}}, \quad (4)$$

i.e., the parameter τ_1 plays the same part in the relations of Elata et al., as the parameter v_{*t} in the Meyer relations.

The second parameter α_E , like the parameter α_M in [1], determines the effectiveness of the investigated solutions.

For flows of guaiacum solutions we calculated the quantities $1/\sqrt{\lambda}$ and $\lg(\operatorname{Re}\sqrt{\lambda})$ from Eq. (2); the results are plotted on the graph of Fig. 6. The relaxation time for these solution was calculated according to Eq. (3), and the parameter α_E was calculated for each value of the concentration on the basis of data on the measured velocity profiles according to the following (from [2]):

$$\alpha_E = \frac{\frac{v}{v_*} - 5.5 - 5.75 \lg \frac{yv_*}{v_s}}{\lg \frac{\tau_1 v_s}{d^2}}. \quad (5)$$

The function $\alpha_E(c)$ thus calculated is shown in Fig. 7a for flows of guaiacum solutions.

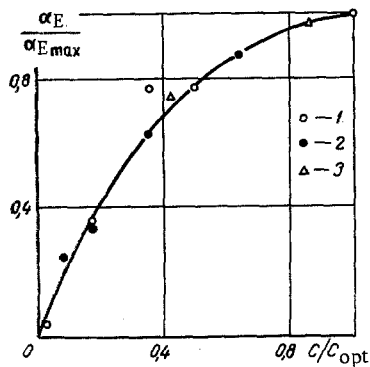


Fig. 8. Experimental curve of the coefficient $\alpha_E/\alpha_{E_{max}}$ versus the reduced concentration c/c_{opt} for flows of polymer solutions. 1) Guaiacum; 2) polyacrylamide; 3) polyox.

It is apparent from the graph of Fig. 6 that relation (2) well describes the experimental results, and the parameter τ_1 can be used to predict the threshold value of the Reynolds number.

The calculations of the threshold Reynolds number for flows of polyacrylamide and polyox solutions also exhibit satisfactory agreement with the experimental data. However, we did not perform measurements of the velocity profiles for these polymers, and it was therefore impossible to carry out a complete calculation according to (2), on account of the lack of data on the function $\alpha_E(c)$.

The law (2) was presumed to be valid for flows of polyox and polyacrylamide solutions, because the experimental results from the drag measurements provide a good fit, in the coordinates $1/\sqrt{\lambda}$ vs $\log(Re\sqrt{\lambda})$, to a straight line, which has a point in common with the analogous line for water in the vicinity of the threshold Reynolds number. The results of a calculation of the parameter $\alpha_E(c)$ according to Eq. (2) for polyacrylamide and polyox solutions are given in Fig. 7b and 7c.

The effectiveness of the polymer solutions, as characterized by α_E , can be expressed independently of the type of polymer and in a universal fashion in the coordinates $\alpha_E/\alpha_{E_{max}}$ vs c/c_{opt} (see Fig. 8).

Inasmuch as the threshold frictional stress at the tube wall does not harbor a large scatter of results for solutions of one type of polymer, the calculations according to Meyer's relation (1) also yield good agreement with the experimental data when the precision of the critical stress is improved on the basis of the results of experiments using solutions of different polymers ($\tau_t = 5$ dyn/cm² for polyox solutions and $\tau_t = 20$ to 25 dyn/cm² for polyacrylamide and guaiacum solutions). The coefficients α_M are calculated in the same way as the coefficients α_E .

We offer the following conclusions based on our experimental data and the foregoing calculations:

1. In the turbulent flow of solutions of the investigated polymers in pipes, a drag-reduction effect is observed when a certain threshold value of the wall temperature is attained. The threshold value of the frictional stress is 5 dyn/cm² for polyox solutions and 20 to 25 dyn/cm² for guaiacum and polyacrylamide solutions.

2. For flows of polyox solutions it is possible to find an optimum value of the concentration $\sim 3 \cdot 10^{-5}$ g/cm³, above which any further increase in the concentration does not increase the drag-reduction effect.

In the experimental range of concentrations of the polyacrylamide and guaiacum solutions the optimum concentration was not reached. However, it may be postulated on the basis of the graphs of Fig. 7a and 7b that this concentration is in the vicinity of $3.5 \cdot 10^{-4}$ g/cm³ and 10^{-3} g/cm³ for the respective polymers.

3. The maximum values of the drag reduction for flows of polymer solutions relative to the case of water was 82% in our experiments for polyox solutions and 70% for guaiacum and polyacrylamide solutions.

4. The graphs of Fig. 7a-7c enable one to calculate the potential drag reduction according to Eq. (2) for flows of polyox, polyacrylamide, and guaiacum solutions of various concentrations in tubes.

NOTATION

d	is the tube diameter;
y	is the distance from tube wall;
c	is the concentration by weight of polymer solution;
c_{opt}	is the optimum concentration by weight of polymer solution;
τ	is the frictional stress;
ν	is the viscosity of water;
ν_s	is the viscosity of solution;
ρ	is the density of water;
q	is the volumetric flow rate of liquid in the tube;
v_s	is the average flow velocity of liquid in the tube;

v	is the flow velocity of liquid in the tube at a distance y from the wall;
$v_* = \sqrt{\tau_w/\rho}$	is the dynamic velocity;
v_{*t}	is the threshold value of dynamic velocity;
$Re = v_s d / \nu_s$	is the Reynolds number;
Re_t	is the threshold value of Reynolds number;
$\lambda = 8\tau_w / \rho v_s^2$	is the drag coefficient;
λ_w	is the drag coefficient for water;
λ_s	is the drag coefficient for solution;
λ_t	is the threshold value of drag coefficient;
α_M	is the Meyer parameter [1];
α_E	is the Elata (et al.) parameter [2];
$\alpha_{E_{max}}$	is the maximum value of α_E ;
τ_1	is the relaxation time;
k	is the Boltzmann constant;
T	is the temperature, °K;
N	is the Avogadro number;
M	is the molecular weight of polymer.

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